sumed or removed from the surface before collision. A more realistic model would necessarily recognize that the propellant consists of a dense three dimensionally packed array of nonuniform oxidizer particles with most of the aluminum contained in "cusp-shaped" binder pockets bounded by surrounding oxidizer particles. In these pockets the aluminum is protected on the binder surface from oxidation by transpiration of binder products, and further protected by the presence of an oxide coating that prevents ignition until very high temperatures are reached. Thus the theoretical model is deficient in its picture of the geometry of the diffusional situation, and even more deficient in its picture of wandering, burning aluminum droplets looking for partners. It seems much more likely that agglomeration is concurrent with ignition of accumulated, oxide coated particles, and that agglomerate size is governed by the microscopic heterogeneity of the propellant. In any case, there is ample evidence in the literature that the aluminum particles will not oxidize at propellant surface temperature at a rate governed by oxidizer diffusion as presumed in the model. We do not contend that this "argument" disproves the model proposed in Ref. 1 and 2, but rather raises a number of issues that would have to be resolved before the model merited acceptance. We feel the issues cannot be resolved in favor of the model, and will seek to establish this point when recent work is fully documented.

In summary, we believe that the results reported in Ref. 1 have doubtful relevance to combustion in rocket motors, are derived from a sampling procedure that does not warrant the conclusions reached, and are correlated by a theoretical model that is so unrealistic as to lack known qualitative attributes of the combustion process. The "reasonable agreement" between experiment and theory is considered to be little more than rationalization.

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Comment on "Buckling of a Cylindrical Shell Loaded by a Pre-Tensioned Filament Winding"

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Nomenclature

D = plate flexural stiffness, $Et^3/12(1 - \mu^2)$
E = Young's modulus
f = function
k = elastic spring constant
k_y = buckling coefficient
k_{yp} = $f(\beta, Z)$ as defined in Ref. 1
K = spring stiffness parameter
L = length of cylindrical shell
m = number of longitudinal waves
n = number of circumferential waves
r = radius of cylindrical shell
t = thickness of cylindrical shell
Z = curvature parameter
β = $nL/\pi r$
μ = Poisson's ratio

Mikulas and Stein[1] are to be congratulated for their interesting note on the subject of shell stability with pre-tensioned filament winding. The result of this note undoubtedly is useful for the optimization study of an integrated system in the conceptual design. However, there are three points not yet discussed in the same note.

1) The minimization of the negative root of the characteristic equation [Ref. 1, Eq. (14)] may also determine the critical stress resultant. This operation can be performed by letting

$$\frac{\partial k_y}{\partial \beta} = \frac{\partial}{\partial \beta}\left\{\frac{k_{yp} - [k_{yp}{}^2 + (4k_{yp}K/\beta^2)]^{1/2}}{2}\right\} = 0 \qquad (1)$$

It seems that Eq. (1) may lead to a different result. It can be shown that [Eq. (14)][1]

$$\frac{k_y{}^2\beta^2}{k_y\beta^2 + K} = \frac{(1+\beta^2)^2}{\beta^2} + \frac{12Z^2}{\pi^4}\frac{1}{(1+\beta^2)^2\beta^2}$$

A quadratic in k_y can be reduced to Eq. (B5)[2]

$$k_y = \frac{(m^2+\beta^2)^2}{\beta^2} + \frac{12Z^2m^4}{\pi^4\beta^2(m^2+\beta^2)^2}$$

which is linear in k_y, if and only if there is no spring stiffness K vanishing in Eq. (14)[1] and m is taken to be unity in Eq. (B5).[2] Similarly their solutions can be reconciled if treated as below. If both roots of Eq. (14)[1] are considered, then Eq. (15)[1] becomes

$$k_y = \{k_{yp} \pm [k_{yp}{}^2 + (4k_{yp}K/\beta^2)]^{1/2}\}/2$$

Then if K vanishes,

$$k_y = \begin{cases} 0 \\ k_{yp} = \dfrac{(1+\beta^2)^2}{\beta^2} + \dfrac{12Z^2}{\pi^4}\dfrac{1}{\beta^2(1+\beta^2)^2} = \\ \end{cases}$$

Eq. (B5)[2] for $m = 1$

It indicates that only the positive root has significance for $K = 0$.

Therefore, for $K \neq 0$, Eq. (14)[1] is not identical to Eq. (B5)[2]; it may be necessary to establish further the uniqueness

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of the critical stress obtained by the minimization of the positive root only while there are two roots available.

2) The conclusion drawn by Eq. (17)[1] may not be universal. Since the insulation materials used on the cryogenic tanks usually are not linear, and the supporting arrangement may not be regular under thermal effect, then the spring stiffness can be either irregular and/or nonlinear. Thus the power law of $\frac{1}{2}$ may not always be valid. By the process of minimization described in Ref. 1, it can be expected that the particular β is a function of Z and K or $\beta_p = f(Z, K, \text{const})$. By the substitution of β_p in Eq. (15),[1] the critical stress is obtained as

$$(k_y)_{\text{crit}} = f(Z, K, \text{const})$$

where

$$Z = (L/r)^2(r/t)(1 - \mu^2)^{1/2}$$

$$K = [12(1 - \mu^2)/E\pi^4](L/r)^4(r/t)^3rk$$

For a given shell geometry, (L/r), (r/t), and r values are fixed. The material properties E and μ also may be assumed invariant by the selection of an arbitrary material for the cylindrical shell. Thus Z is held constant in this case, and the determination of k_y hinges on the form of k (the elastic spring constant generally approximated by the insulation properties and the supporting arrangement, as calculated by Fig. 2, Ref. 1). The importance of a wider class of k forms to confirm the power law thus derived may be of definite interest. However, the difficulties in obtaining more data are anticipated, since the job of winding the pre-tensioned filament on the insulation of extra light weight is quite a formidable task by itself.

3) The influence of a minimized β value has been recognized in both Refs. 1 and 2. They state that, at low values of Z, buckling is characterized by a large number of circumferential waves. As Z increases, the number of circumferential waves decreases. This of course agrees with the definition of $\beta \equiv (n/\pi)(L/r)$. For a fixed value of β, n is inversely proportional to (L/r). However, the critical value of n is not shown in the theory given in Ref. 1. It may prove to be of interest to minimize the roots of Eq. (14)[1] by performing $\partial^2 k_y/\partial n \partial (L/r) = 0$ in the hope of defining the critical n value for a (L/r) or vice versa, if this information is ever needed.

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Errata: "Plastic Buckling of Axially Compressed Cylindrical Shells"

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IN the above article 1) the right-hand side of Eq. (21) should be divided by $3^{1/2}$ and 2) the isolated $2 - \nu$ in front of the square brackets in the last of Eqs. (27) should be deleted.

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